

ASYMPTOTIC BUILDING ENVELOPE

Combining the benefits of asymptotic and principal curvature layouts

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Abstract. This paper presents current research on low-cost, high-performance doubly curved building envelopes that benefit from a curvilinear layout along an isothermal web on minimal surfaces. We investigate the geometric symbiosis of an elastic lamella mullion system that follows the asymptotic curves combined with panelization strategies along the principal curvature lines in order to simplify fabrication and enhance the structural performance. We present a digital and physical setup of a full-scale prototypical asymptotic façade substructure. In collaboration with the construction industry, we evaluate the complete design and construction process, including digital modeling, fabrication planning, computer-aided manufacturing and logistics, prefabrication, and assembly. Both digital and physical prototypes are used to investigate cladding solutions. Our studies are specifically looking at developable-elastic and planar-rigid tessellations, which utilize a principal curvature layout. We conclude by highlighting flat-sheet fabrication, high structural resilience, and assembly time and tolerances, as primary potentials and challenges of this design strategy.

Keywords. Asymptotic Curves; Principal Curvature Line; Double Curvature; Building Envelope; Elastic Construction.

1. Introduction

There is an urgent need for innovative building construction methods that combine high structural efficiency with design freedom and integrate well into the built environment. Great potential lies in double-curved systems, such as gridshells. These form-active structures enable a spatial load transfer via compression and tension, allowing for optimal use of material to create lightweight, transparent building envelopes. Nonetheless, their application in architecture remains rare and specialized, as their free-form geometry creates high costs in the fabrication and assembly of individual and spatially complex parts. To achieve a shell-load-transfer, a consistent double curvature and tangential supports are

necessary, which constraints the design and often lacks to integrate with the urban environment.

The research branch of **Architectural Geometry** (Pottmann et al. 2015) has produced fundamental insights on topology optimization for curved structural grids (Bo et al. 2011; Bartoň et al. 2013; Pellis and Pottmann 2018) as well as curved building skins (Liu et al. 2006; Huard et al. 2015; Eversmann et al. 2016) with the goal to simplify fabrication and allow the use of planar or developable building elements.



Figure 1. Asymptotic lamella gridshells. A: The Inside/Out Pavilion at the Technical University in Munich. B: The Intergroup Hotel Canopy in Ingolstadt.

Recent research on elastic gridshells has presented **asymptotic curves (AC)** (following the path of vanishing normal curvature), as beneficial network for lamella construction, as they facilitate simple fabrication from straight and flat elements with repetitive orthogonal nodes (Schling 2018). Asymptotic lamella networks allow for a simple, self-forming erection process, where the weak axis of the lamellas is elastically bent and twisted to form the design shape, while the strong axis creates high resilience against external loads (Schikore et al. 2019). This system thus combines the structural benefits of a gridshell and grillage, offers a versatile design from curved to flat, and allows smooth integration into the urban environment. However, this design method has only been used for free-standing sculptural projects without a cladding solution (Figure 1). The potential for integrated building skins has not been investigated.



Figure 2. Double-curved facade paneling following the principal curvature directions. A: The Strasbourg train station. B: The Eiffel Tower Pavilions. C: The Schubert Club Band Shell.

In opposition to the asymptotic layout, research on doubly curved building skins has promoted **principal curvature lines (PCL)** (following the max/min normal curvature), which encode the path of vanishing geodesic torsion and thus allow rectangular, planar-quad (PQ) and developable panels (Pottmann et al. 2008; Pottmann et al. 2010; Tang et al. 2016). Such methods have been applied in the

construction of building envelopes (Figure 2), f.e. looking at cylindrical glass panels (Strasbourg train station, Eiffel Tower Pavilion) or planar quadrilateral panels (The Schuber Club Band Shell).

This paper presents a novel construction system for doubly curved curtain wall systems, which combines a structural layout along the AC with a cladding layout along the PCL. Both networks are combined in an **isothermal web on a minimal surface** (Pottmann et al. 2007, p. 648) bisecting each other and thus creating reciprocal benefits for structural bracing and façade connections. We first present our digital design method for a curtain wall corner module. We introduce a novel construction strategy and describe the manufacturing, prefabrication, assembly and erection process, highlighting the benefits and challenges of elastic, asymptotic construction. Finally, we present three cladding solutions driven by the principal curvature direction.

2. Digital Method

Both ACs and PCLs create quadrilateral networks, but never align with each other. There is, however, a geometric sweet spot in the form of minimal surfaces in which both networks form almost quadratic cells and can be defined with the same intersection points. Such a layout is called an **isothermal web**. (Due to the tight paper-restriction of 10 pages max, the computational method to model minimal surfaces, ACs and PCLs in an isothermal web was omitted from this paper. More details can be found in the dissertation of Eike Schling (Schling 2018)).

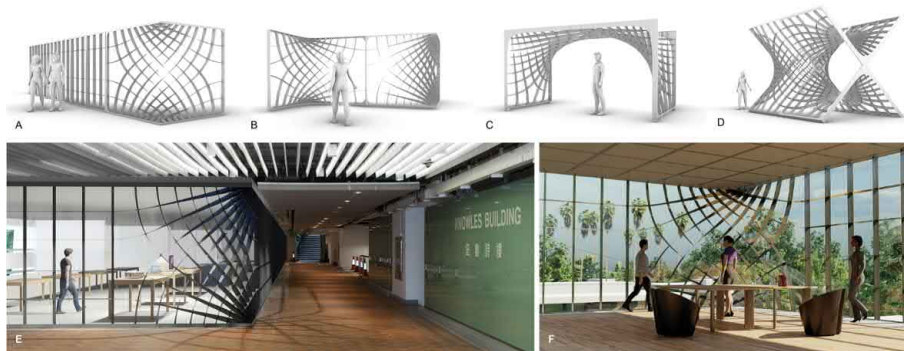


Figure 3. Asymptotic Scherk module. The corner module allows smooth integration into existing mullion and transom systems (A), and may be combined to create modular building envelopes (B and C) or even periodic continuous network typologies (D). We anticipate integrating this construction technology into the built environment to allow a smooth transition from a standard, curtain-wall systems into a diagonal, doubly curved layout.

Modular design. The Scherk singly periodic cell was used as basis for our façade prototype (Figure 5), as it can be positioned as a corner module into any rectangular building type (A), but also be combined to create multiple architectural scenarios, in which modules tie smoothly into one another (B and C) or even create a periodic minimal surface network (D). The façade module exhibits a

smooth transition from a straight and regular transom-and-mullion-typology to the distinctive, diagonal, and twisted asymptotic network. We aim to take advantage of this transition to integrate a curved expressive corner module in a standard curtain wall system (E, F).

3. Asymptotic Substructure

The constructive development of the substructure builds upon knowledge from previous projects (Figure 1), looking at double-lamella bolted construction (A) (Schling et al. 2018), and single lamella welded construction (B) (Schikore et al. 2020). Both systems work with a slotted connection to interlace two families of lamellas within one level. This strategy allows for a simple assembly of the lamellas as flat segments on the ground and a subsequent deformation into the designed doubly curved shape. Once the final geometry is generated, the joints are fixed in their final 90-degree-position. There is room for improvement: Handling the single strips of steel is precarious, as the slots create weak spots, which are prone to develop kinks during assembly. Once the structure is completed, the lamellas are weakest between two joints in their lateral direction. Here, we can observe a buckling behavior along the lines of Euler Case 2 (hinged on top and bottom), where the slotted joints act as hinges, resulting in a buckling length similar to the joint-to-joint distance ($s_k = h$). This behavior has a direct effect on the buckling stress and deflections under compression.

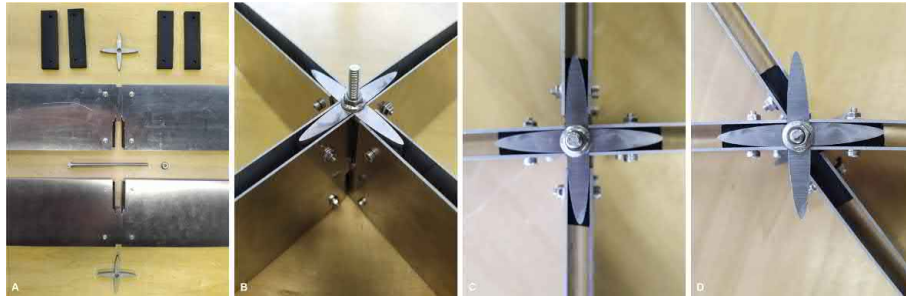


Figure 4. Composite joint system from two 1.5mm stainless steel strips with 8mm rubber inlays (A). The joints are designed to allow up to $+30^\circ$ rotation freedom during assembly (D) and lock at 90 degrees once the rounded star-washers are fixed (B, C).

3.1. JOINT SYSTEM

The construction development was focused on creating more resilient lamellas for the assembly and the load-bearing structure. The novel joint system is created from two stainless steel strips of 1.5 mm (and 100 mm height) and rubber interlays of 8 mm, and is prefabricated into composite elements with 11 mm thickness (Figure 8). The rubber reinforces the slotted steel to prevent kinks and allow for a continuous elastic behaviour. Within the joints, the rubber-and-steel-composite acts like cartilage-and-bone-system, in which the rubber is a flexible mediator between two intersecting lamellas and allows for up to $\pm 30^\circ$ rotation of the scissor

joints (Figure 8, D). This is necessary to facilitate the transformation of the lamella grid from flat to curved geometry. Once the final shape is created, the star-shaped washers are fastened to enforce the 90° angle, while the rounded sides of the stars still allow for continuity in curvature (Figure 8, C). This reinforcement through rubber and star-washer aims to create a rigid connection, thus shifting the buckling mode towards Euler Case 4 (clamped on top and bottom) and reducing the buckling length to 0.5 ($s_k = 0.5h$).

3.2. PLANNING, FABRICATION AND LOGISTICS

All strips are fabricated straight and flat with a standardized joint detail. The only information needed from the 3D model is the distances between joints and the chamfer-angles at the end of each strip to fit the outer steel frame. Their geodesic curvature creates a slight variation in length between the inner and outer strip within one lamella (Figure 9). Therefore, the joint distances are measured individually for each strips. To produce the 2D drawings, we simply draw straight parallel strips, mark the intersections, copy the standardized slot-detail to each intersection marker, and finally add the chamfered ends. The 3D lamellas are modeled and unrolled as control geometry.

The double symmetric network of our design has only five different lamella geometries. There are two families of lamellas, one on the bottom, with slots pointing up, and one on top, with slots pointing down, and each lamella consists of two metal strips. This creates a total number of 20 individual strips, each produced twice. They are labelled with a 3-digit-system by number (1-5), family (top or bottom) and position within the lamella (left or right). Laser-cutting and shipping benefit greatly from the straight geometry of lamellas, creating almost no offcuts of the rectangular stainless steel sheet material and minimal packing size (Figure 10).

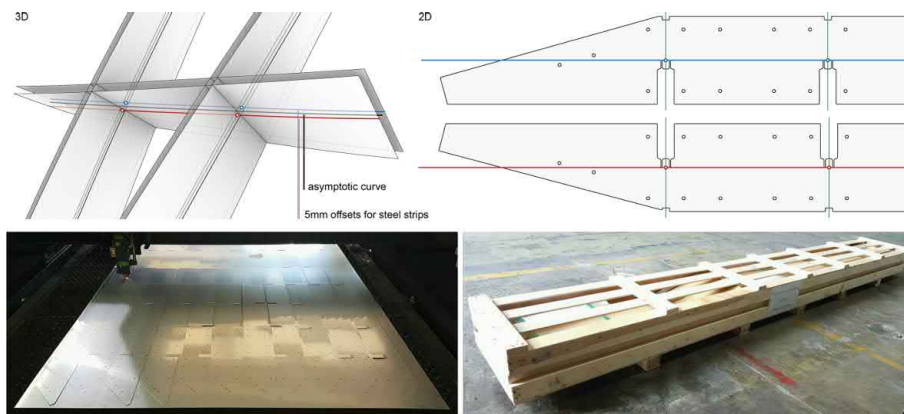


Figure 5. Fabrication. Top: The joint-to-joint distance and chamfered edges are measured individually for each metal strip. Bottom: The straight steel lamellas can be cut from sheet metal with virtually no waste material and packed and shipped efficiently.

3.3. CONSTRUCTION

The construction process consists of prefabrication of lamellas, assembly of the grid, deformation of the grid, fastening of the joints, and finally, fixture within the rigid frame.

The composite-lamellas are **prefabricated** by hand in the workshop (Figure 11, A, B). Two rubber pieces are placed left and right of each slot-joint and fastened with two rivets each. This process embeds the geodesic curvature into the lamellas and makes them resilient to kinks. The grid of lamellas is **assembled** flat on the ground (C, D). The star-washers are not yet fastened, as the joints need to be able to accommodate up to 30° rotation. The assembled grid is placed on a cross-shaped frame and **elastically formed** into its curved design shape (E-H) without the need for form-work. The final geometry is confirmed simply by measuring the diagonal distance at the top and bottom. This deformation adds geodesic torsion within the lamellas. Once the final geometry is completed, the **star-washers are fastened** to fix the 90° angle and align the lamellas into one level. This process was most challenging, as lamellas and stars had to be aligned with some force, and the bolts had to overcome high friction in the joints. Finally, the curved grid is bolted to the **rigid steel frame** (K-L), which acts as a supporting beam. The final module weighs approximately 300 kg, including the 6 mm steel frame. It is used in the following research as substructure for different panelization strategies.

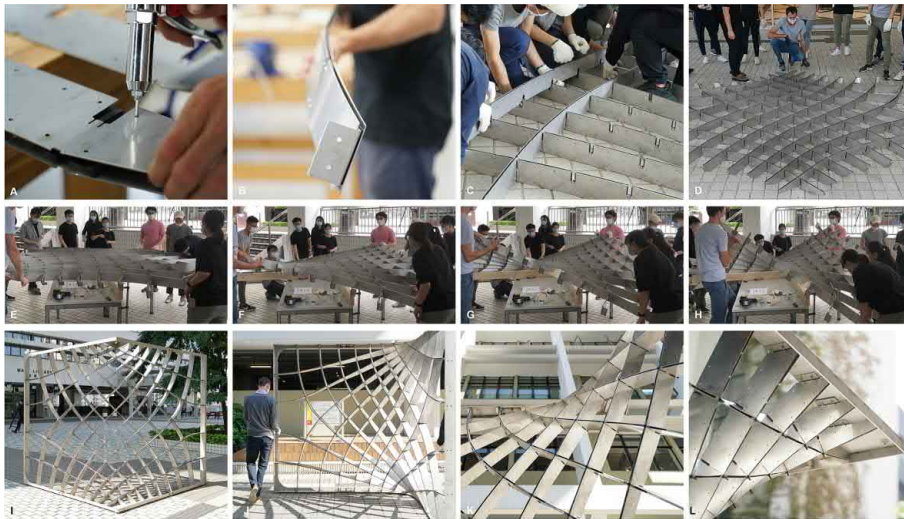


Figure 6. A-D: Prefabrication. E-H: Elastic formation. I-L: Completed module.

4. Double Curved Envelope Strategies

The digital and physical substructure was used to investigate multiple cladding options (Figure 12). In this section, we will focus on triangulated, planar-quad, and developable strips, as they take geometrical advantage of the isothermal web.

4.1. TRIANGULATED FAÇADE

The triangulated typology visualizes clearly the possible reading of layouts (Figure 13). It can be created with horizontal (A) or vertical (B) folds resulting in regular joints, supporting six adjacent glass panels. Such panelizations read as discrete strips similar to the developable façade (4.3). An alternative approach is to alternate horizontal and vertical folds (C), creating alternating joints with either 4 or 8 glass panels. Here, the effect of our isothermal network becomes visible, as the clusters of 4 triangles naturally lie within one plane and read like a planar quad. We propose a simple point bracket for frameless glazing with silicon joints (D).

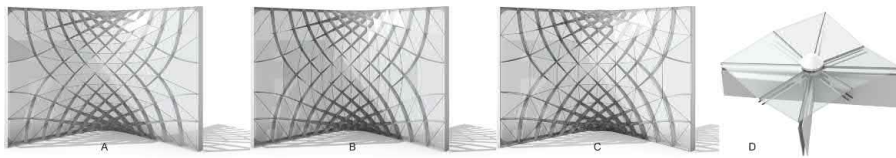


Figure 7. Three possible triangular layouts, along the horizontal (A), vertical (B) or alternating (C) directions.

4.2. PLANAR QUADS

The principal curvature directions encode the path of vanishing geodesic torsion. A quadrilateral mesh along these directions will consist of planar faces (Figure 14). The isothermic network creates nearly square panels which allows efficient fabrication (minimal offcuts) and a homogenous visual pattern. The panels are positioned diagonally to the asymptotic layout (A). They are fixed at four joints and cover a fifth. As the asymptotic lamellas exhibit no normal curvature, their joints are planar so that the x-shaped lamella intersection neatly nestles with the panel above. We fabricated a proof-of-concept for the prototype using quasi-square acrylic glass panels (B). An alternative fixing system was developed by Marilies Wedl (Wedl 2020) at the Technical University in Vienna. Wedl proposes a four point bracket at every alternate joint on a laminated timber structure (C). Her 1:1 prototypical model beautifully illustrates the subtle anticlastic folds between four planar glass panels, following the principal curvatures (D).

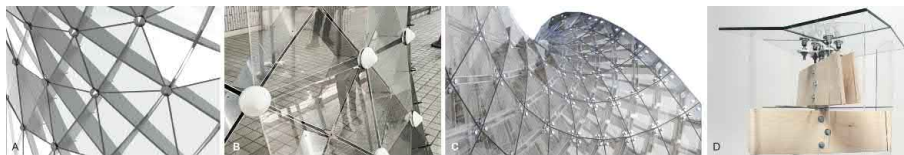


Figure 8. Planar-quad glazing along the principal curvature directions in steel (A,B) and timber (B,C) (Wedl 2020).

4.3. DEVELOPABLE STRIPS

Tangential developable strips are nothing but an infinitely dense set of planar quads (Liu et al. 2006) and may be designed along one family of PC-lines. Such a semi-discrete panelization beautifully translates the design language of elastic strip construction into the building skin (Figure 15). In this case, the panels are not planar but create a singly curved geometry that is either convex (A) (bending away from the substructure) or concave (B) (bending towards the substructure). For a concave layout, an additional offset at each connection joint is needed to prevent the acrylic glass from colliding with the diagonal lamellas. We propose aluminum strips to fix the panels and offer structural bracing through triangulation (D). We fabricated a proof-of-concept (C) for a concave layout, creating a vertical cover.



Figure 9. Developable strip facade along the principal curvature directions.

The physical testing of the cladding strategies revealed tolerances of up to 10 mm between the digital model and the physical prototype. The panels were remeasured and fitted individually.

5. Conclusion

This paper presents a design method for low-cost, high-performance, doubly curved building envelopes. The method relies on the geometric sweet-spot of isothermal webs on minimal surfaces, creating a quasi-square layout of both asymptotic curves and principal curvature lines bisecting each other at 45° . We design a versatile cubic 2.4 m prototype that can be integrated into existing buildings to create smooth transitions from curved to a flat curtain wall system. The asymptotic grid is used as the steel lamella substructure. This allows for a simple fabrication of slender, flat and straight steel strips, producing minimal offcuts. We prefabricate single-curved, composite lamellas using rubber inlays to improve the elastic and structural performance during assembly and load-bearing state. After flat assembly, the lamella grid is deformed into double curvature and fixed to the rigid rectangular steel frame. The slender lamellas enable an elastic erection while maintaining high resilience against normal loads, such as wind, typical for curtain walls. Finally, both digital and physical models are used to develop cladding strategies along the principal curvature lines, diagonal to the asymptotic substructure. We give an overview of options and present a triangulated, planar square and elastic developable system, comparing their functional and aesthetic quality, offsets and fixtures.

5.1. INSIGHTS

The isothermal design combines simple fabrication and assembly, and high structural resilience for the asymptotic substructure, with geometric benefits, namely planar-quad and developable layouts, for the cladding. The cubic prototype presented here is a first attempt to digitally and physically combine these qualities and test the fabrication workflow for both substructure and cladding. The main challenges of this technique are the tolerance and friction which arise from an elastic construction. Tolerances specifically need to be kept at a minimum to allow for accurate high quality cladding.

5.2. FUTURE RESEARCH

The computational tool to draw asymptotic and principal curvature lines are well developed. The isothermal web, however, is created individually by hand. An automated process would speed up the design process immensely. The fabrication and assembly of the steel substructure were carried out over the course of approx. 8 hours. Optimizing the details of the slot to avoid friction of rubber and steel might speed up the assembly and fixing process. Furthermore, we anticipate testing other materials, like GRP and aluminum, to lower the total weight of the façade module. The strategies for cladding are yet in an experimental design stage. Here, a further investigation of material, scale, tolerances, maintenance and functionality in weather conditions is of utter importance.

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References

- Barthel, R. 2005, Natural Forms - Architectural Forms, in W. Nerdinger (ed.), *Frei Otto - Complete Works. Lightweight construction - natural design*, Birkhäuser, Basel, 16–30.
- Bartoň, M., Shi, L., Kilian, M., Wallner, J. and Pottmann, H.: 2013, Circular Arc Snakes and Kinematic Surface Generation, *Computer Graphics Forum*, **32**(2pt1), 1-10.
- Bauer, A.M. and Längst, P.: 2019, “Kiwi3d! Meshfree, Isogeometric FE Analysis integrated in CAD”. Available from Chair of Structural Analysis, Technical University Munich<<https://www.kiwi3d.com/>> (accessed 27.05.2020).

- Bo, P., Pottmann, H., Kilian, M., Wang, W. and Wallner, J.: 2011, Circular arc structures, *ACM Transactions on Graphics*, **30** (101)(DOI: 10.1145/1964921.1964996), 1-11.
- Eversmann, P., Schling, E., Ihde, A. and Louter, C.: 2016, Low Cost Double Curvature. Geometrical and Structural Potentials of Rectangular, Cold-bent Glass Construction, *K. Kawaguchi, M. Ohsaki, T. Takeuchi (Eds.): IASS Annual Symposium 2016*, Tokyo.
- Huard, M., Eigensatz, M. and Bompas, P.: 2015, Planar Panelization with Extreme Repetition, *Philippe Block, Jan Knippers, Niloy J. Mitra, Wenping Wang (Eds.): Advances in Architectural Geometry 2014*, London, 259–279.
- Kiendl, J.: 2011, *Isogeometric Analysis and Shape Optimal Design of Shell Structures*, Ph.D. Thesis, Chair of Structural Analysis, Technical University Munich.
- Liu, Y., Pottmann, H., Wallner, J., Yang, Y.L. and Wang, W.: 2006, Geometric modeling with conical meshes and developable surfaces, *ACM Transactions on Graphics*, **25** (3)(DOI: 10.1145/1141911.1141941), 681.
- Oberbichler, T.: 2019, “Bowerbird. Plug-In for Grasshopper” . Available from Chair of Structural Analysis, Technical University Munich<<https://github.com/oberbichler/Bowerbird>> (accessed 19.11.2020).
- Pellis, D. and Pottmann, H.: 2018, Aligning principal stress and curvature directions, *Lars Hesselgren, Karl-Gunnar Olsson, Axel Kilian, Samar Malek, Olga Sorkine-Hornung, Chris Williams (Eds.): Advances in Architectural Geometry 2018*, Gothenburg, 34-53.
- Pottmann, H., Asperl, A., Hofer, M. and Kilian, A.: 2007, *Architectural Geometry*, Springer & Bentley Institute Press, ISBN: 978-3-211-99765-9.
- Pottmann, H., Eigensatz, M., Vaxman, A. and Wallner, J.: 2015, Architectural Geometry, *Computers and Graphics*, **47**(DOI: 10.1016/j.cag.2014.11.002), 145-164.
- Pottmann, H., Huang, Q., Deng, B., Schiftner, A., Kilian, M., Leonidas, G. and Wallner, J.: 2010, Geodesic Patterns, *ACM Trans. Graphics*, **29** (4)(DOI: 10.1145/1778765.1778780), 1-10.
- Pottmann, H., Schiftner, A., Bo, P., Schmiedhofer, H., Wang, W., Baldassini, N. and Wallner, J.: 2008, Freeform surfaces from single curved panels, *ACM Transactions on Graphics*, **27**(76)(DOI: 10.1145/1360612.1360675), 1-10.
- Schikore, J., Bauer, A.M., Barthel, R. and Bletzinger, K.U.: 2019, Large torsion on elastic lamella grid structures, *Carlos Lázaro, K.-U. Bletzinger, Eugenio Oñate (Eds.): FORM and FORCE 2019*, Barcelona, 807-814.
- Schikore, J., Schling, E., Bauer, A.M. and Oberbichler, T.: 2020, Kinetics and Design of Semi-Compliant Grid Mechanisms, *Olivier Baverel, Helmut Pottmann, Caitlin Mueller, Tomohiro Tachi (Eds.): Advances in Architectural Geometry 2020*, Paris, (to be published).
- Schling, E.: 2018, *Repetitive Structures. Design and Construction of Curved Support Structures with Repetitive Parameters*, Ph.D. Thesis, Chair of Structural Design, Technical University of Munich (DOI: 10.14459/2018md1449869).
- Schling, E., Hitrec, D. and Barthel, R.: 2017, Designing Grid Structures using Asymptotic Curve Networks, *Klaas de Rycke, Christoph Gengnagel, Olivier Baverel, Jane Burry, Caitlin Mueller, Minh Man Nguyen et al. (Eds.): Design Modelling Symposium 2017*, Paris, 125-140.
- Tang, C., Kilian, M., Bo, P., Wallner, J. and Pottmann, H.: 2016, Analysis and design of curved support structures, *Sigrid Adriaenssens, Fabio Gramazio, Matthias Kohler, Achim Menges, Mark Pauly (Eds.): Advances in Architectural Geometry 2016*, Zurich, 8-23.
- Wedl, M.: 2020, *Ein Beitrag zur Erstellung einer Gitterschale unter der Verwendung asymptotischer Kurven auf Minimalflächen*, Master’s Thesis, Technischen Universität Wien, Vienna. Fakultät für Architektur und Raumplanung.